

Solving Problems in Physics

- All of the **Problem-Solving Strategies** and **Examples** in the textbook will follow these four steps:
- **Identify** the relevant concepts, target variables, and known quantities, as stated or implied in the problem.
- **Set Up** the problem: Choose the equations that you'll use to solve the problem, and draw a sketch of the situation.
- **Execute** the solution: This is where you “do the math.”
- **Evaluate** your answer: Compare your answer with your estimates, and reconsider things if there's a discrepancy.

Estimation

“Back of the Envelope Calculations”

- Estimate the average area available to each person
 - in the USA.
 - In the World.
- IDENTIFY:
 - We need:
 - Approximate number of people in the USA
 - Approximate number of people in the World
 - Approximate area of the USA
 - Approximate area of the world.
 - Execute:
 - We only care about the orders of magnitude.

Lesson 2

Tooling up:

- Vectors

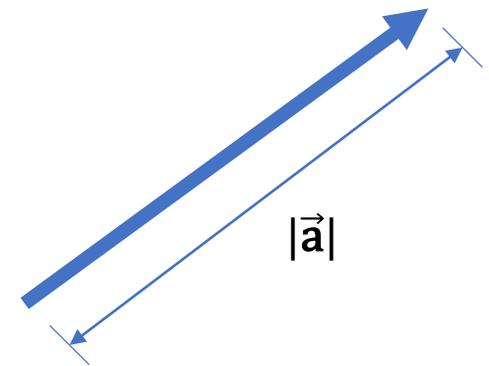
Vectors

- Physical quantities are either:
 - Scalar quantities: only magnitude.
Eg: Mass, Temperature, Frequency.
 - Vectors quantities: magnitudes and direction.
Eg: Displacement, Velocity, Force.

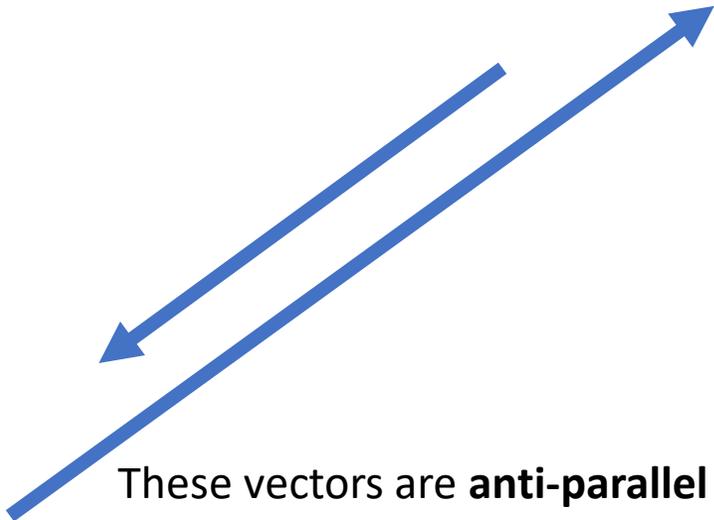
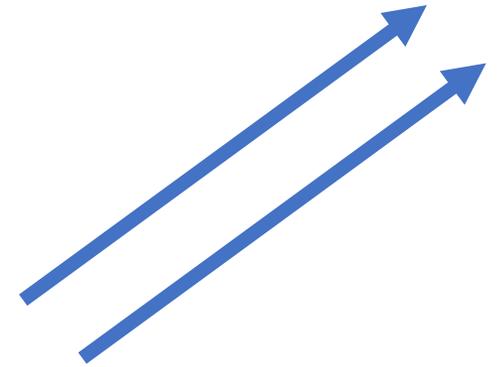
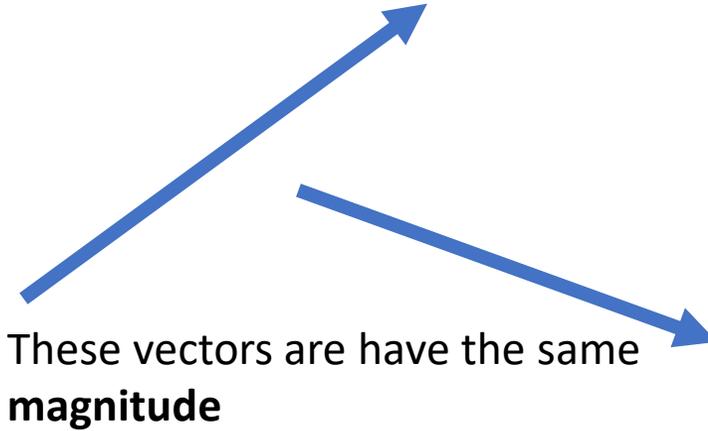
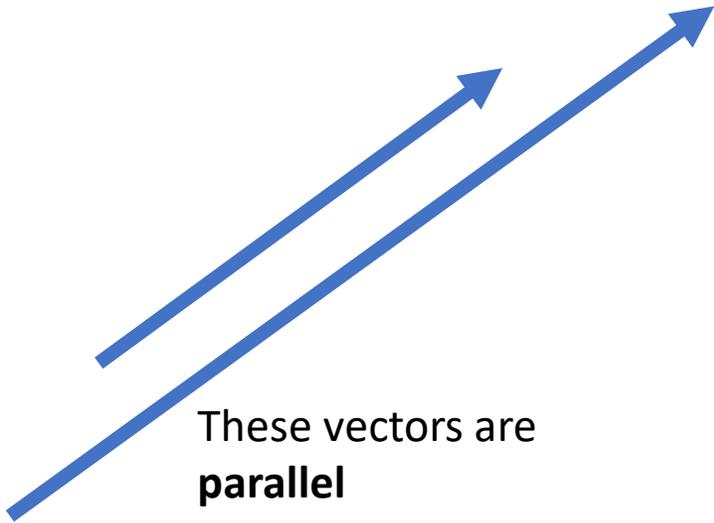
We represent vectors as an arrows.

Algebraically we write them with an arrow \vec{a} .

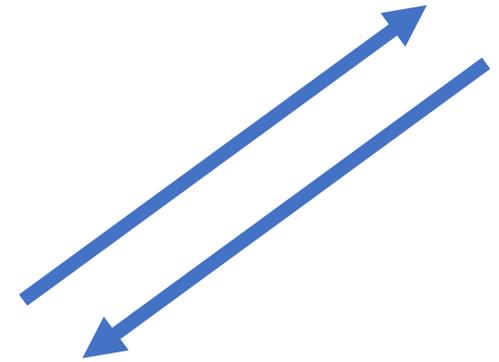
The magnitude is denoted by: $|\vec{a}|$



Vector relations

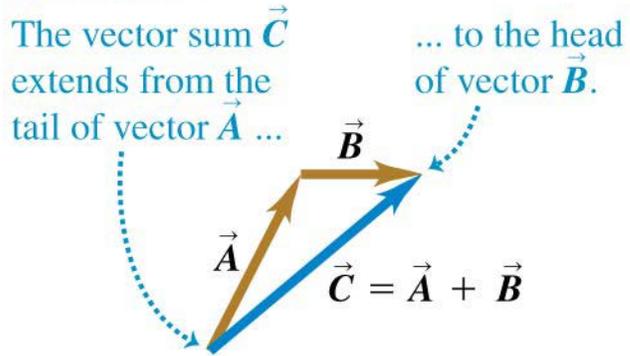


Two vectors are equal when they have the same magnitude and direction.

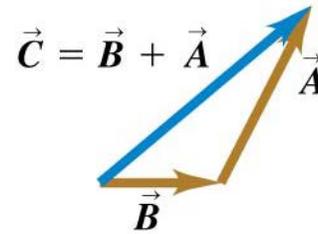


Vector addition (graphic representation)

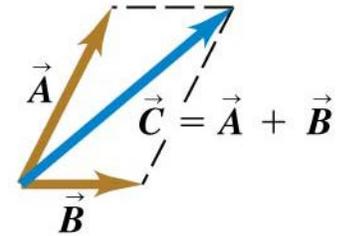
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.



(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.

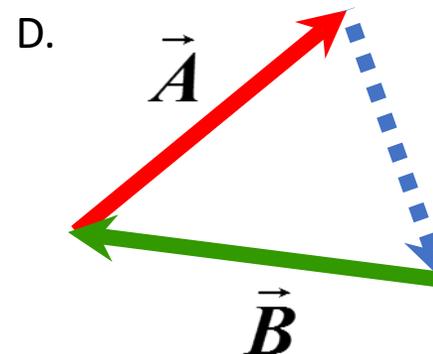
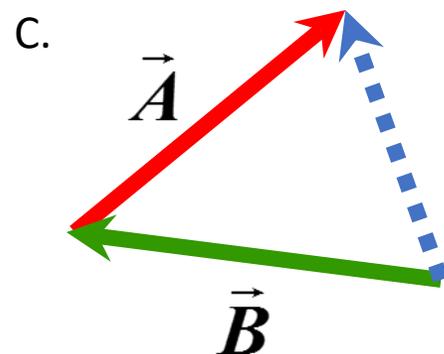
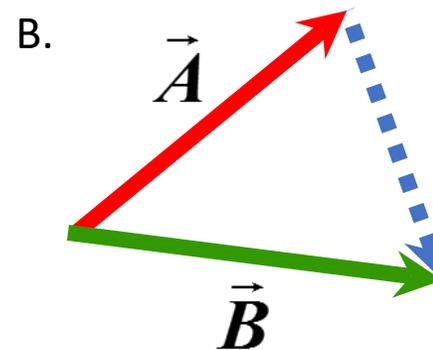
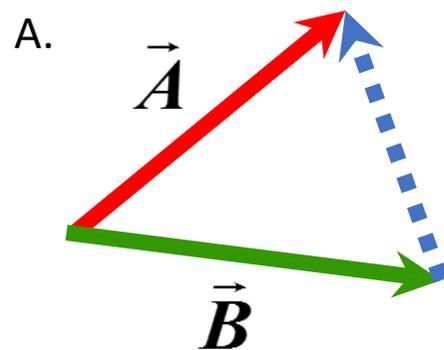


Vector addition is Commutative

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Q1.4

In which case is the blue dashed vector equal to $\vec{A} + \vec{B}$?

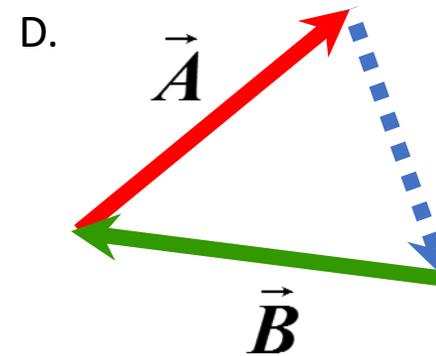
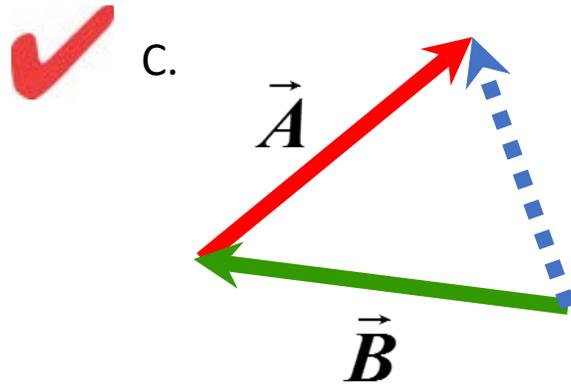
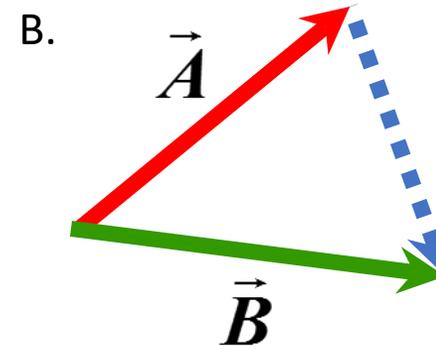
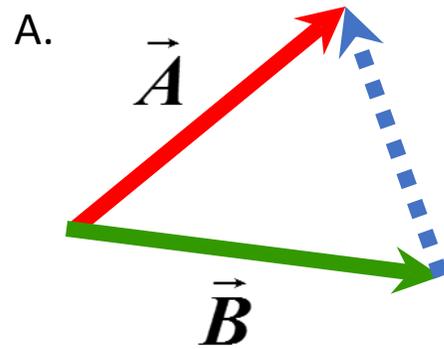


E. Not enough information is given to decide.

A1.4

In which case is the blue dashed vector equal to

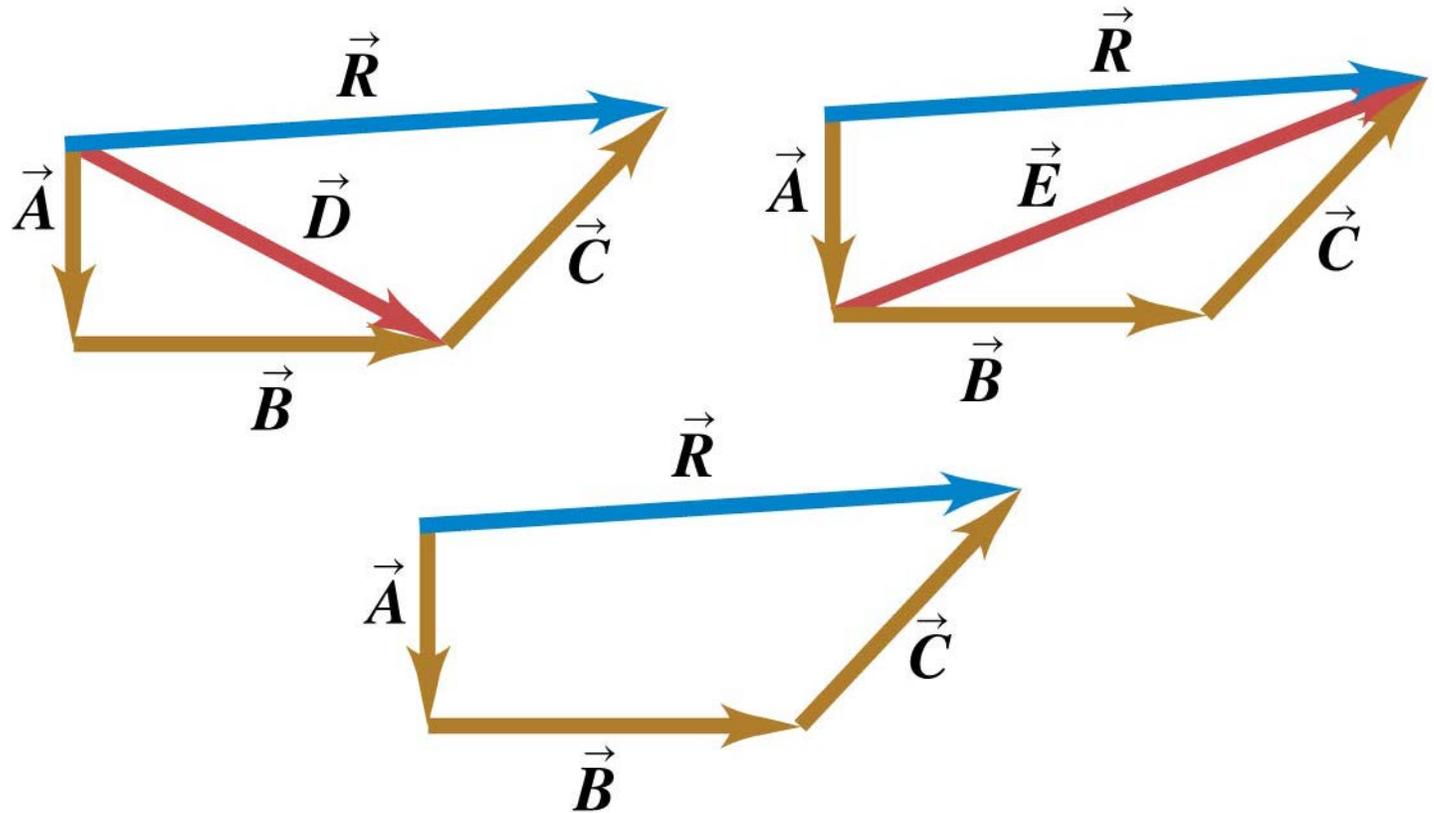
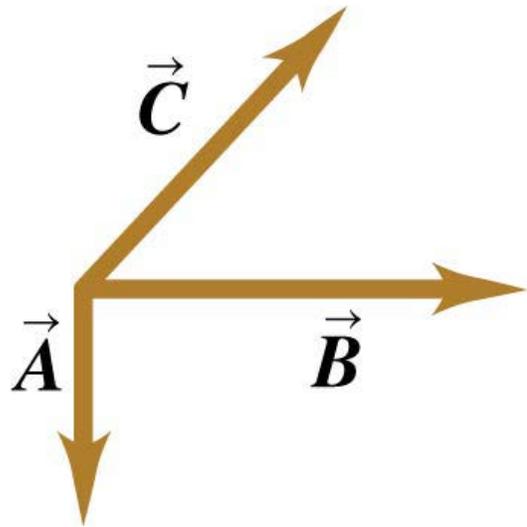
$$\vec{A} + \vec{B}?$$



E. Not enough information is given to decide.

Vector addition is associative

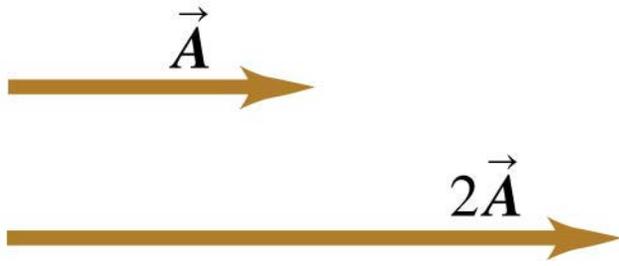
(a) To find the sum of these three vectors ...



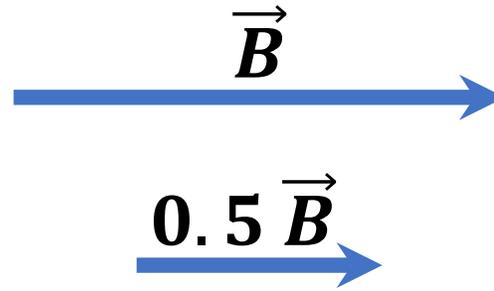
Vector addition is Associative

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) = \vec{a} + \vec{b} + \vec{c}$$

Multiplying a vector by a scalar (Graphically)



Multiplying by a scalar larger than one stretches the vector.

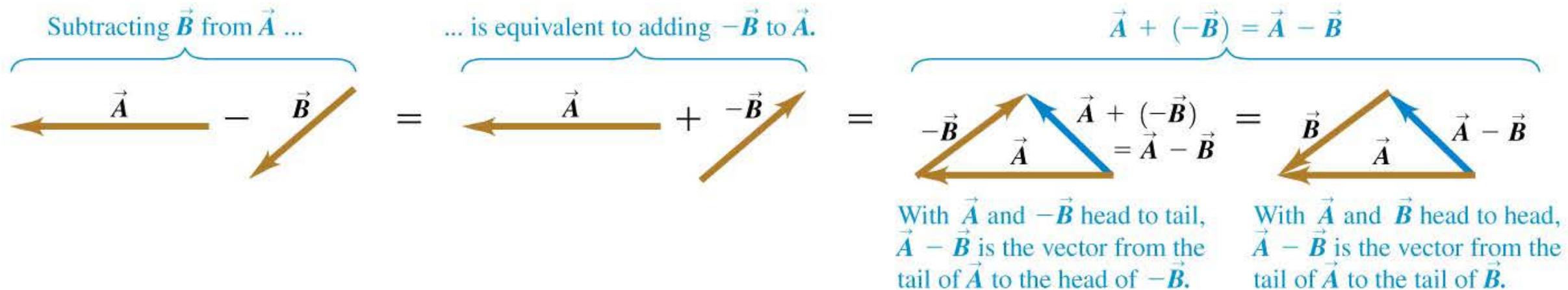


Multiplying by a scalar smaller than one shortens the vector.



Multiplying by a negative scalar reverses the direction vector.

Vector subtraction

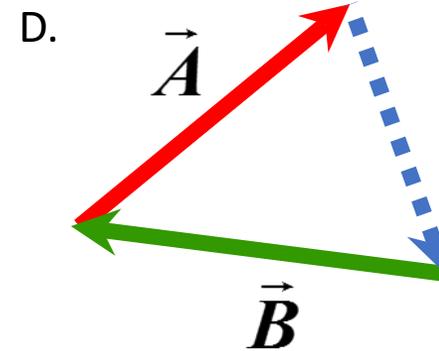
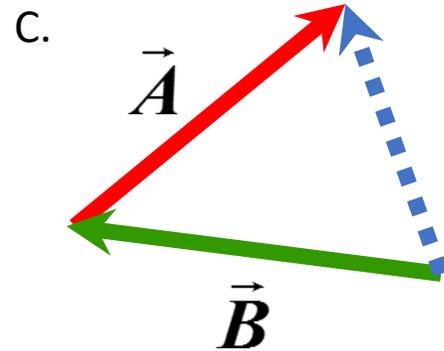
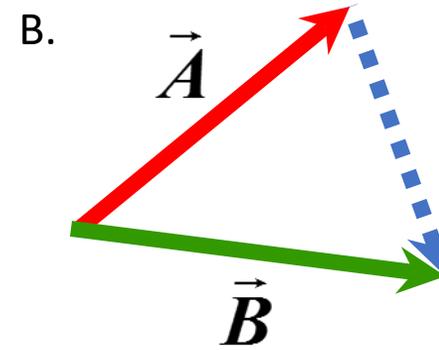
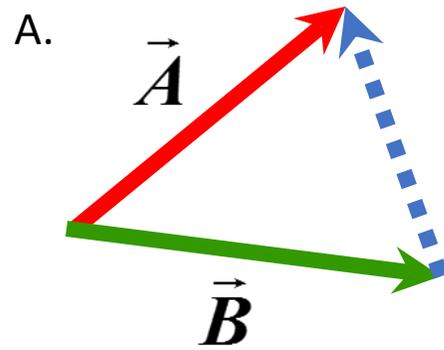


Vector subtraction is NOT Commutative

Vector subtraction is NOT Associative

Q1.5

In which case is the blue dashed vector equal to $\vec{A} - \vec{B}$?



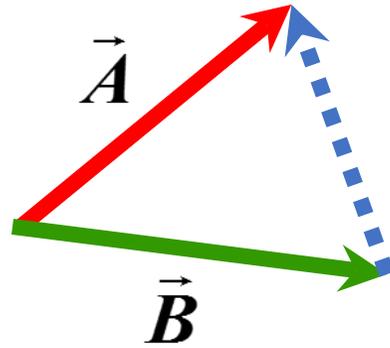
E. Not enough information is given to decide.

A1.5

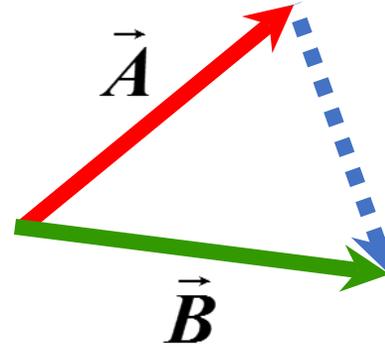
In which case is the blue dashed vector equal to $\vec{A} - \vec{B}$?



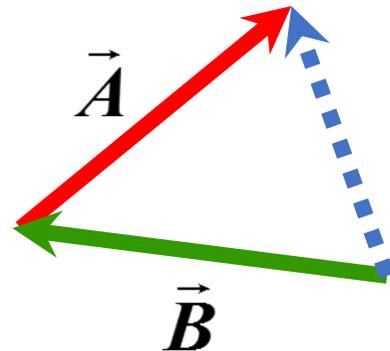
A.



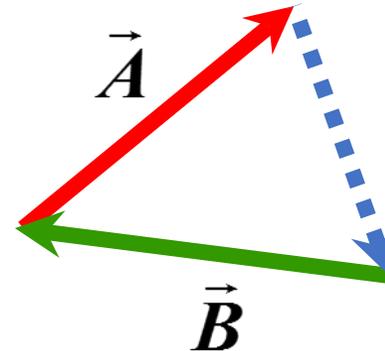
B.



C.



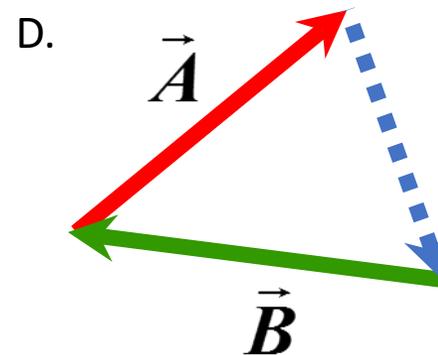
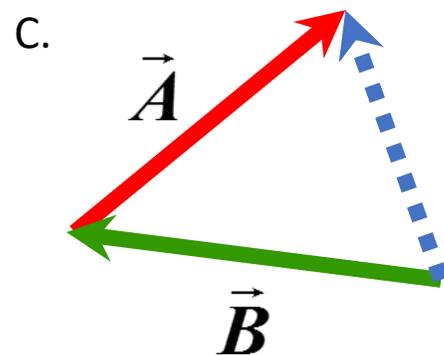
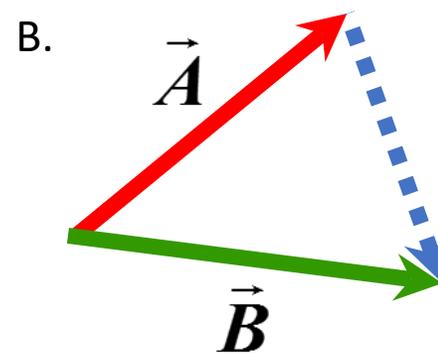
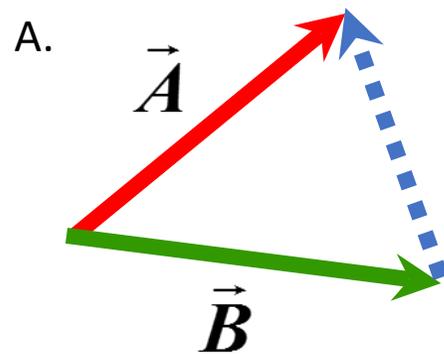
D.



E. Not enough information is given to decide.

Q1.6

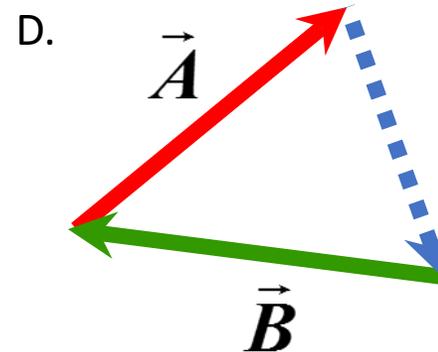
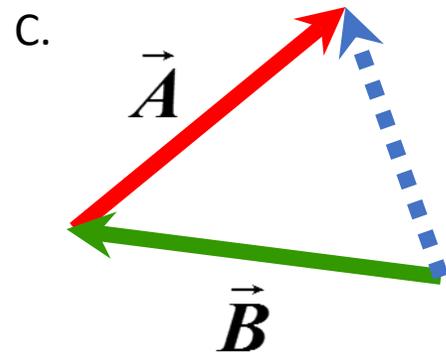
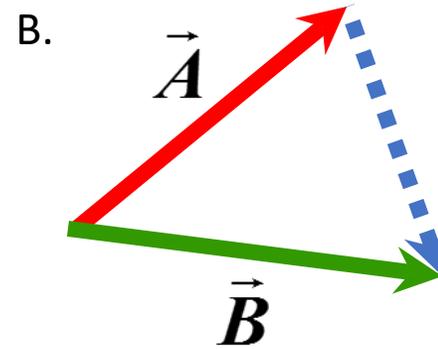
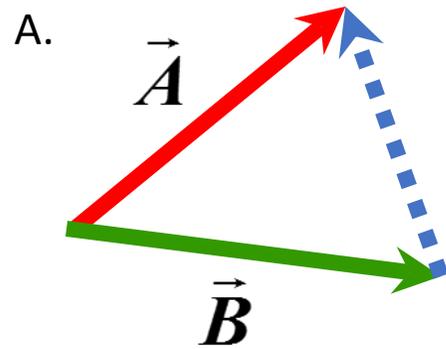
In which case is the blue dashed vector equal to $\vec{B} - \vec{A}$?



E. Not enough information is given to decide.

A1.6

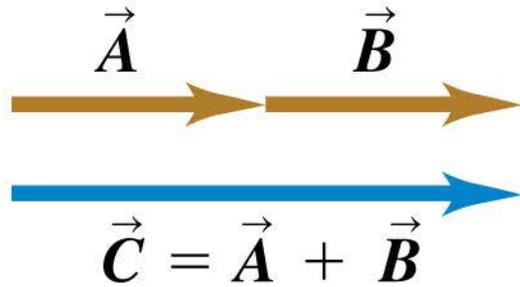
In which case is the blue dashed vector equal to $\vec{B} - \vec{A}$?



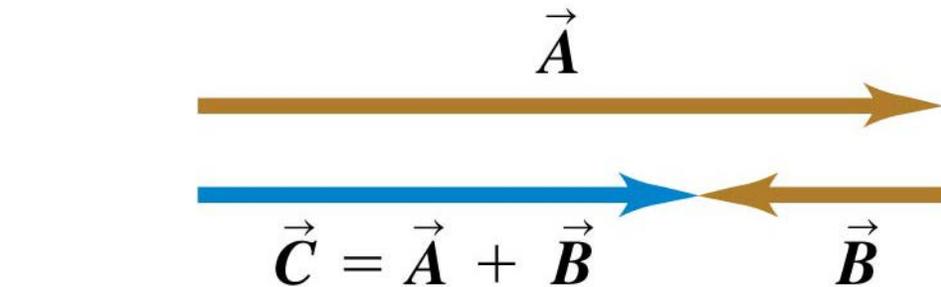
E. Not enough information is given to decide.

Two special cases for vector addition

(a) Only when vectors \vec{A} and \vec{B} are parallel does the magnitude of their vector sum \vec{C} equal the sum of their magnitudes: $C = A + B$.



(b) When \vec{A} and \vec{B} are antiparallel, the magnitude of their vector sum \vec{C} equals the *difference* of their magnitudes: $C = |A - B|$.



Q1.9

Which of the following statements is correct for *any* two vectors \vec{A} and \vec{B} ?

- A. The magnitude of $\vec{A} + \vec{B}$ is $A + B$.
- B. The magnitude of $\vec{A} + \vec{B}$ is $A - B$.
- C. The magnitude of $\vec{A} + \vec{B}$ is greater than or equal to $|A - B|$.
- D. The magnitude of $\vec{A} + \vec{B}$ is greater than the magnitude of $\vec{A} + \vec{B}$.
- E. The magnitude of $\vec{A} + \vec{B}$ is $\sqrt{A^2 + B^2}$.

A1.9

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- A. The magnitude of $\vec{A} + \vec{B}$ is $A + B$.
- B. The magnitude of $\vec{A} + \vec{B}$ is $A - B$.
-  C. The magnitude of $\vec{A} + \vec{B}$ is greater than or equal to $|A - B|$.
- D. The magnitude of $\vec{A} + \vec{B}$ is greater than the magnitude of $\vec{A} + \vec{B}$.
- E. The magnitude of $\vec{A} + \vec{B}$ is $\sqrt{A^2 + B^2}$.

Q1.10

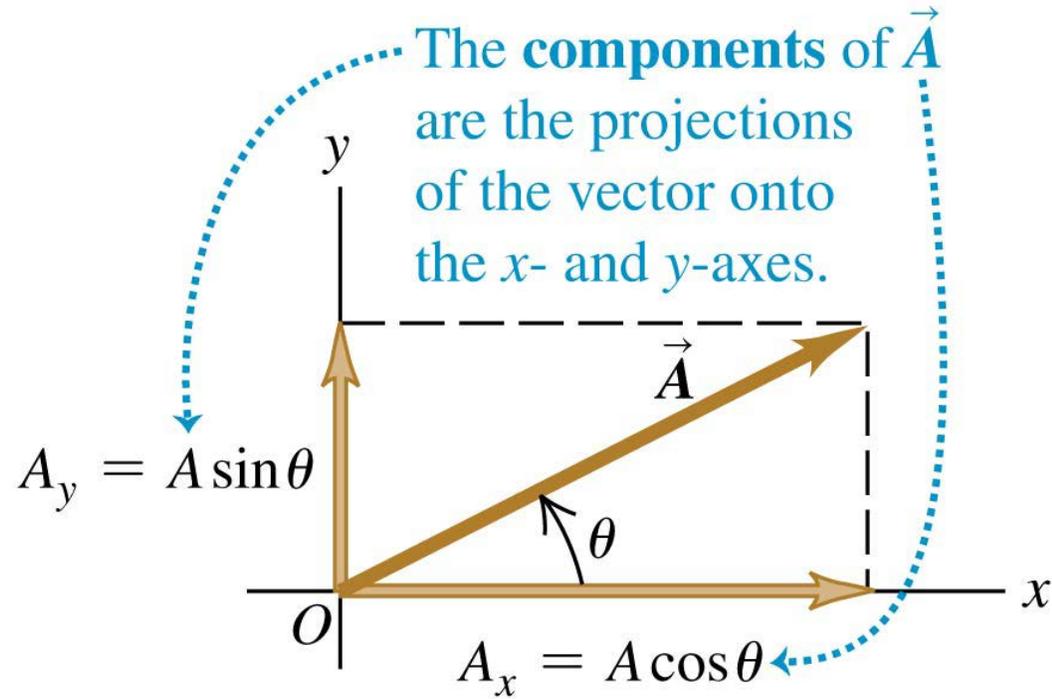
Which of the following statements is correct for *any* two vectors \vec{A} and \vec{B} ?

- A. The magnitude of $\vec{A} - \vec{B}$ is $A - B$.
- B. The magnitude of $\vec{A} - \vec{B}$ is $A + B$.
- C. The magnitude of $\vec{A} - \vec{B}$ is greater than or equal to $|A - B|$.
- D. The magnitude of $\vec{A} - \vec{B}$ is greater than the magnitude of $\vec{A} + \vec{B}$.
- E. The magnitude of $\vec{A} - \vec{B}$ is $\sqrt{A^2 + B^2}$.

A1.10

Which of the following statements is correct for *any* two vectors \vec{A} and \vec{B} ?

- A. The magnitude of $\vec{A} - \vec{B}$ is $A - B$.
- B. The magnitude of $\vec{A} - \vec{B}$ is $A + B$.
-  C. The magnitude of $\vec{A} - \vec{B}$ is greater than or equal to $|A - B|$.
- D. The magnitude of $\vec{A} - \vec{B}$ is greater than the magnitude of $\vec{A} + \vec{B}$.
- E. The magnitude of $\vec{A} - \vec{B}$ is $\sqrt{A^2 + B^2}$.



In this case, both A_x and A_y are positive.

$$\vec{A} = (A_x, A_y)$$

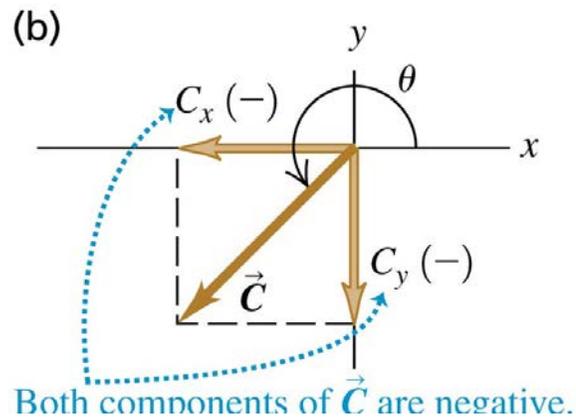
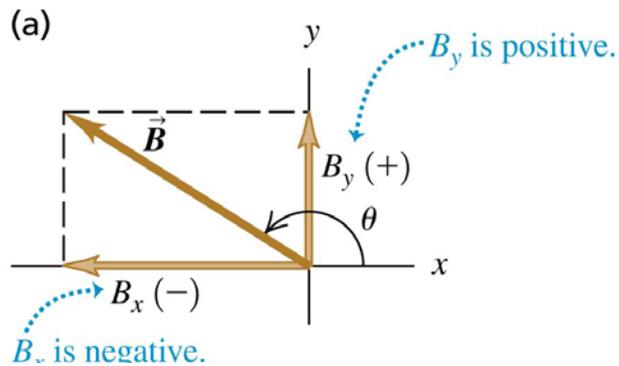
In 2 dimensions.

$$A_x = |\vec{A}| \cos \theta$$

$$A_y = |\vec{A}| \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{a_y}{a_x} \right)$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$



The function

$$\tan^{-1}(x)$$

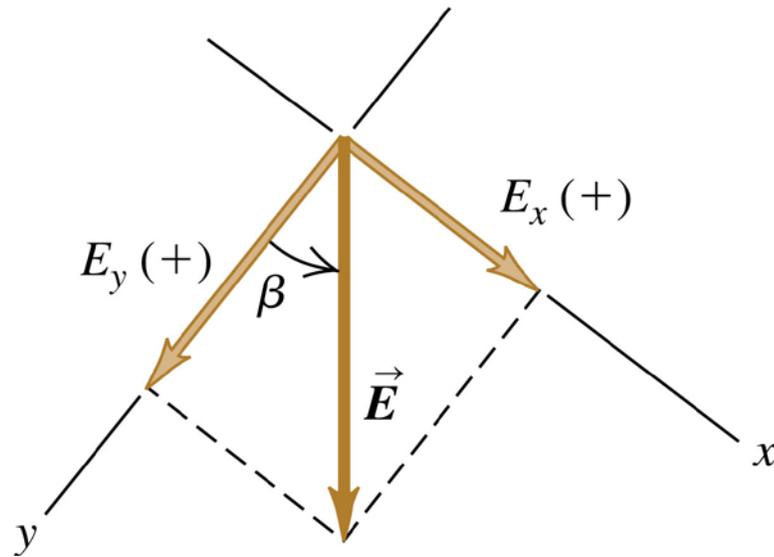
Has as range between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Make sure you get the right quadrant.

Q1.2

What are the x- and y-components of the vector \vec{E} ?

(b)

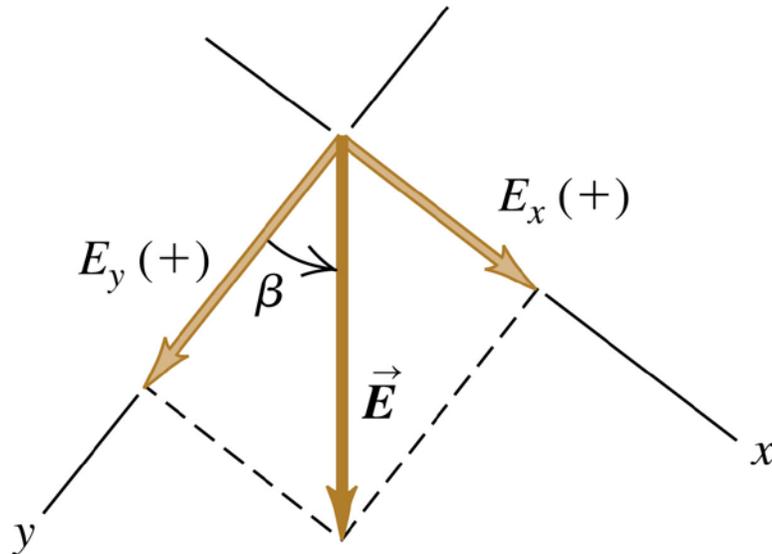


- A. $E_x = E \cos \beta, E_y = E \sin \beta$
- B. $E_x = E \sin \beta, E_y = E \cos \beta$
- C. $E_x = -E \cos \beta, E_y = -E \sin \beta$
- D. $E_x = -E \sin \beta, E_y = -E \cos \beta$
- E. $E_x = -E \cos \beta, E_y = E \sin \beta$

A1.2

What are the x- and y-components of the vector \vec{E} ?

(b)



A. $E_x = E \cos \beta, E_y = E \sin \beta$

B. $E_x = E \sin \beta, E_y = E \cos \beta$

C. $E_x = -E \cos \beta, E_y = -E \sin \beta$

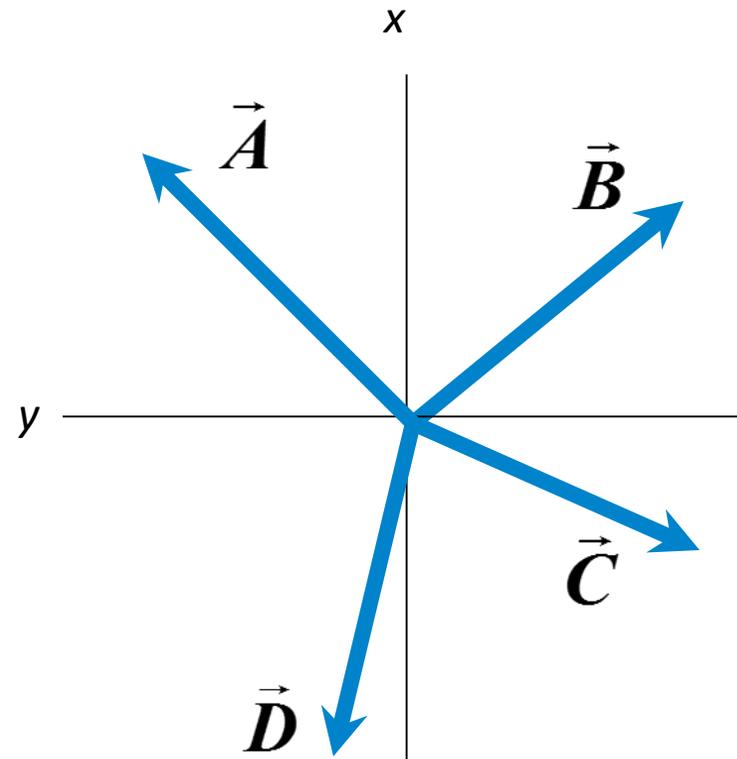
D. $E_x = -E \sin \beta, E_y = -E \cos \beta$

E. $E_x = -E \cos \beta, E_y = E \sin \beta$

Q1.3

Which of these four vectors has a negative x-component and a positive y-component?

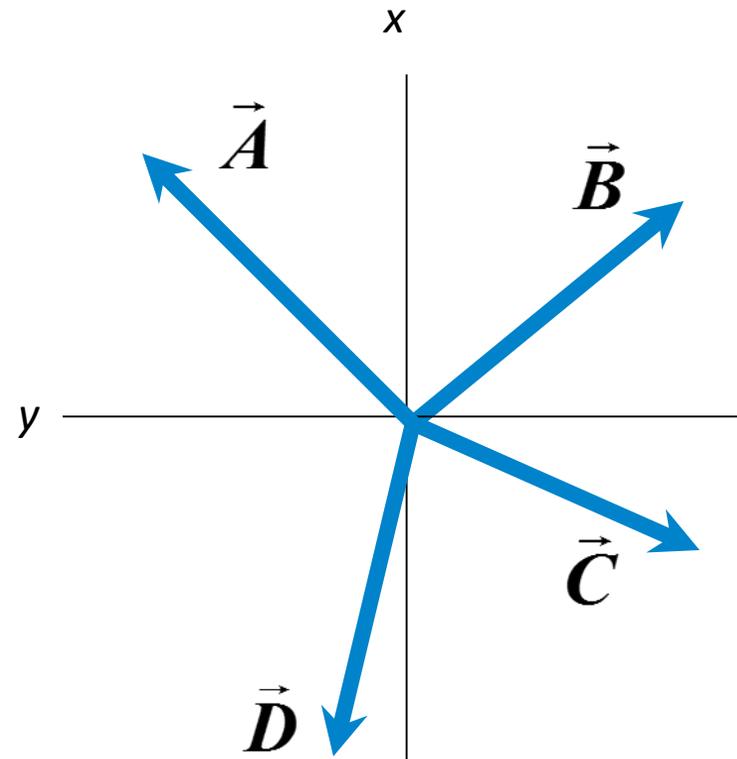
- A. \vec{A}
- B. \vec{B}
- C. \vec{C}
- D. \vec{D}
- E. none of these



A1.3

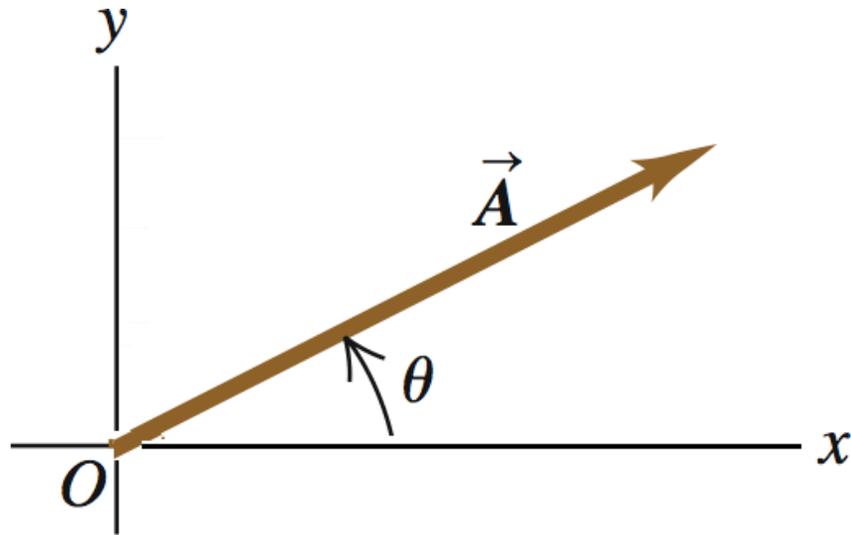
Which of these four vectors has a negative x-component and a positive y-component?

- A. \vec{A}
- B. \vec{B}
- C. \vec{C}
- D. \vec{D}
- E. none of these



Q-R1.1

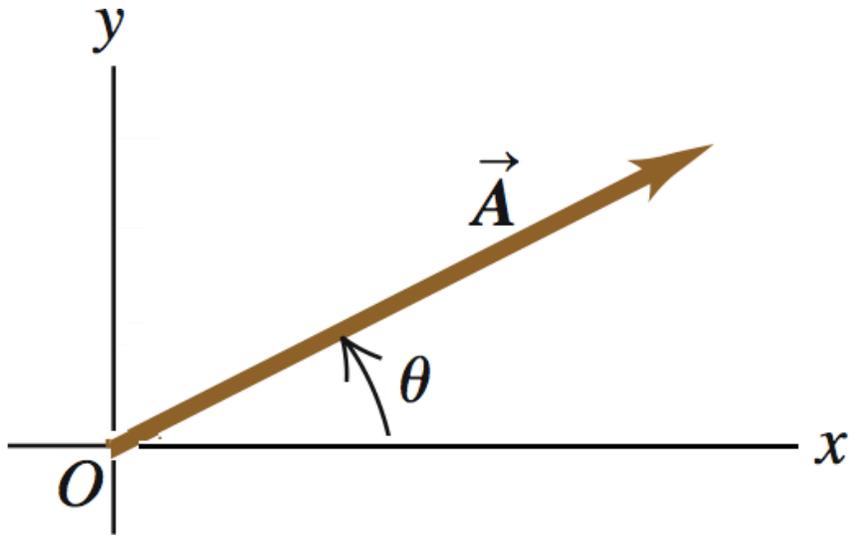
The angle θ is measured counterclockwise from the positive x-axis as shown. **Rank the five vectors listed** in order of their value of θ , from largest to smallest.



- A. $+24\hat{i} + 18\hat{j}$
- B. $-24\hat{i} - 18\hat{j}$
- C. $-18\hat{i} + 24\hat{j}$
- D. $-18\hat{i} - 24\hat{j}$
- E. $+18\hat{i} - 24\hat{j}$

A-RT1.1

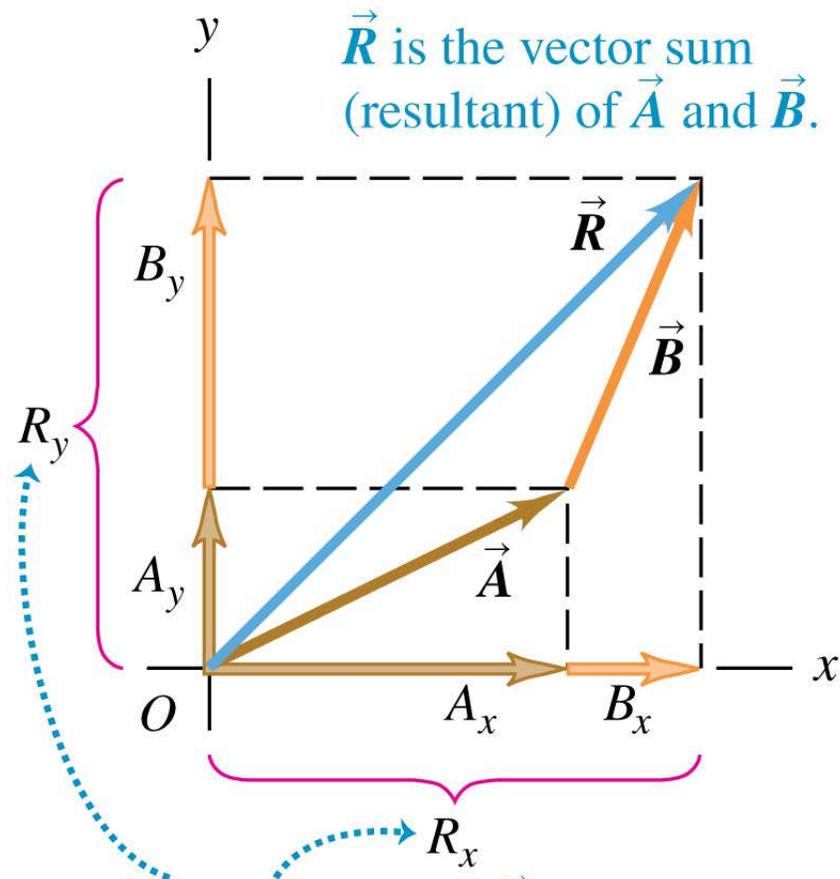
The angle θ is measured counterclockwise from the positive x-axis as shown. **Rank the five vectors listed** in order of their value of θ from largest to smallest.



- A. $+24\hat{i} + 18\hat{j}$
- B. $-24\hat{i} - 18\hat{j}$
- C. $-18\hat{i} + 24\hat{j}$
- D. $-18\hat{i} - 24\hat{j}$
- E. $+18\hat{i} - 24\hat{j}$



Answer: EDBCA

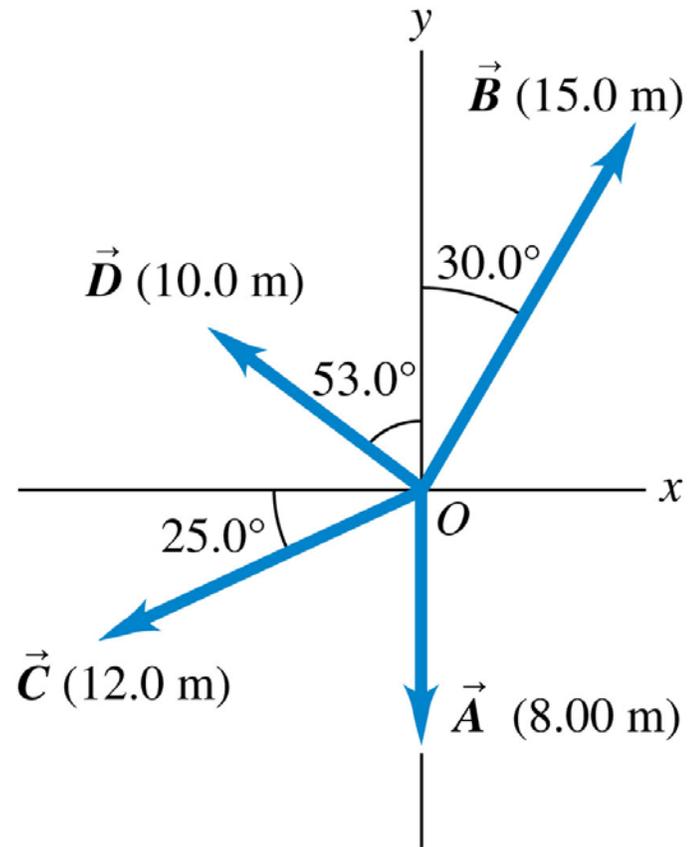


The components of \vec{R} are the sums of the components of \vec{A} and \vec{B} :

$$R_y = A_y + B_y \quad R_x = A_x + B_x$$

Q1.7

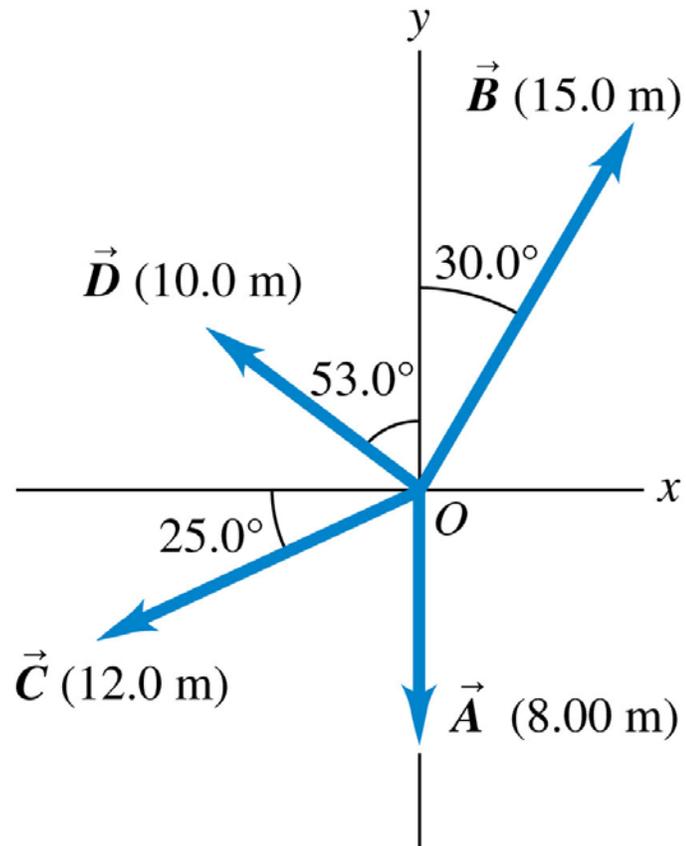
Consider the vectors shown. Which is a correct statement about $\vec{A} + \vec{B}$?



- A. x-component > 0 , y-component > 0
- B. x-component > 0 , y-component < 0
- C. x-component < 0 , y-component > 0
- D. x-component < 0 , y-component < 0
- E. x-component $= 0$, y-component > 0

A1.7

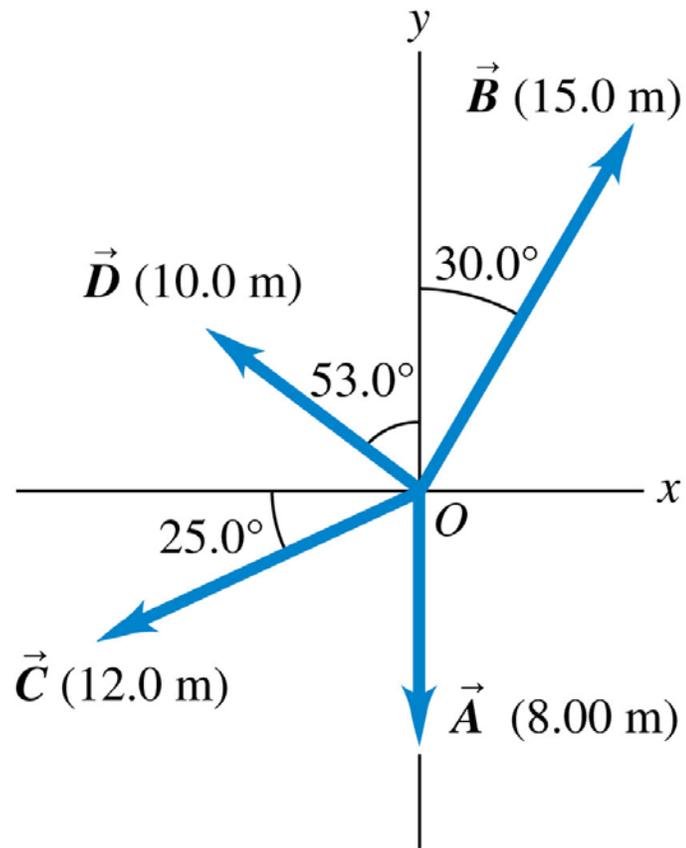
Consider the vectors shown. Which is a correct statement about $\vec{A} + \vec{B}$?



- ✓
- A. x-component > 0 , y-component > 0
 - B. x-component > 0 , y-component < 0
 - C. x-component < 0 , y-component > 0
 - D. x-component < 0 , y-component < 0
 - E. x-component $= 0$, y-component > 0

Q1.8

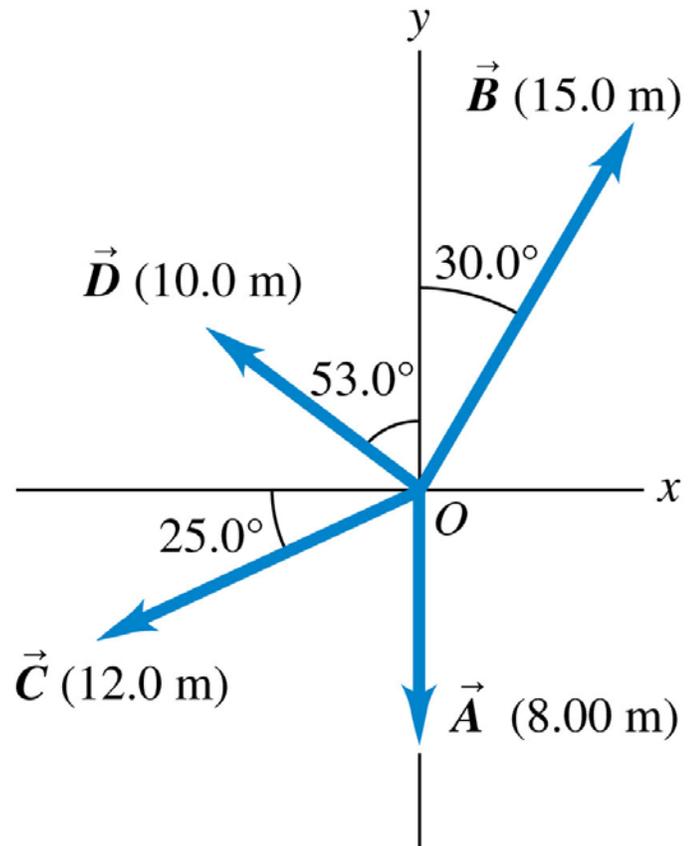
Consider the vectors shown. Which is a correct statement about $\vec{A} - \vec{B}$?



- A. x-component > 0 , y-component > 0
- B. x-component > 0 , y-component < 0
- C. x-component < 0 , y-component > 0
- D. x-component < 0 , y-component < 0
- E. x-component $= 0$, y-component > 0

A1.8

Consider the vectors shown. Which is a correct statement about $\vec{A} - \vec{B}$?



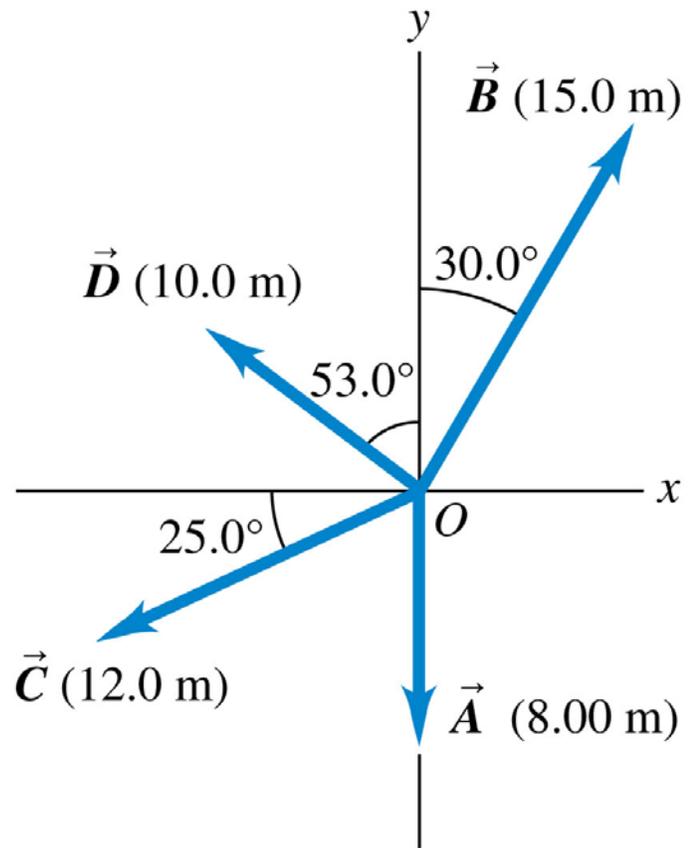
- A. x-component > 0 , y-component > 0
- B. x-component > 0 , y-component < 0
- C. x-component < 0 , y-component > 0
- D. x-component < 0 , y-component < 0
- E. x-component $= 0$, y-component > 0



Q1.11

Consider the vectors shown. What are the components of the vector

$$\vec{E} = \vec{A} + \vec{D}?$$

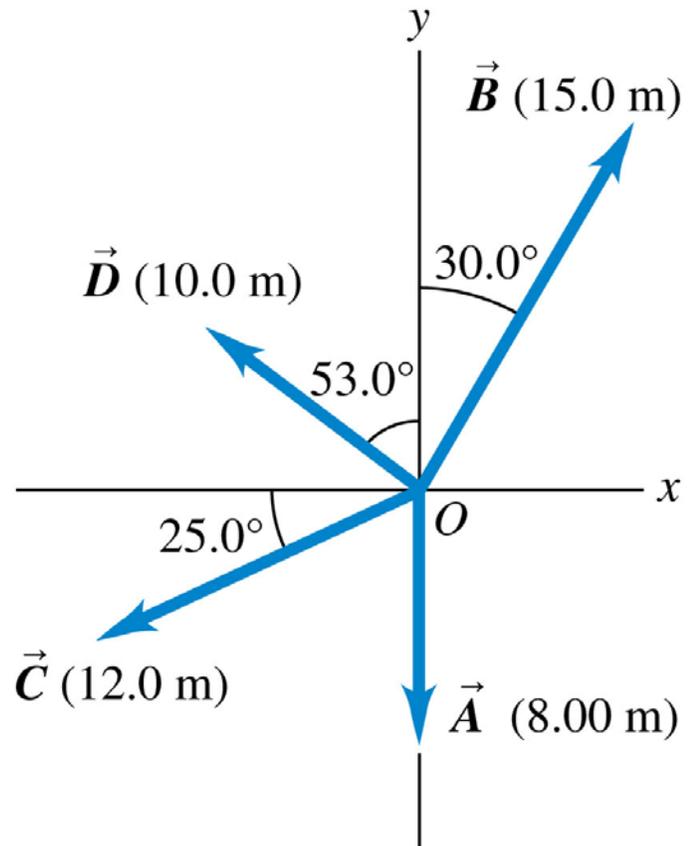


- A. $E_x = -8.00$ m, $E_y = -2.00$ m
- B. $E_x = -8.00$ m, $E_y = +2.00$ m
- C. $E_x = -6.00$ m, $E_y = 0$
- D. $E_x = -6.00$ m, $E_y = +2.00$ m
- E. $E_x = -10.0$ m, $E_y = 0$

A1.11

Consider the vectors shown. What are the components of the vector

$$\vec{E} = \vec{A} + \vec{D}?$$



A. $E_x = -8.00 \text{ m}$, $E_y = -2.00 \text{ m}$

B. $E_x = -8.00 \text{ m}$, $E_y = +2.00 \text{ m}$

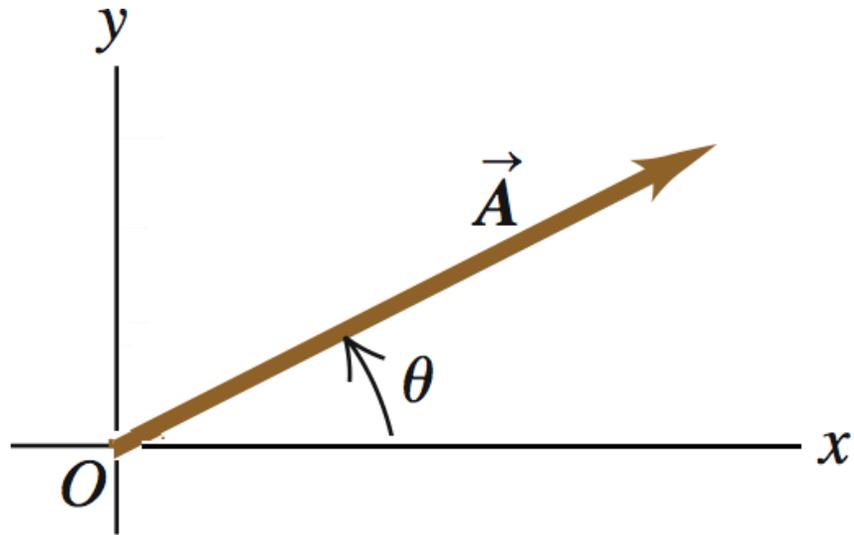
C. $E_x = -6.00 \text{ m}$, $E_y = 0$

D. $E_x = -6.00 \text{ m}$, $E_y = +2.00 \text{ m}$

E. $E_x = -10.0 \text{ m}$, $E_y = 0$

Q1.12

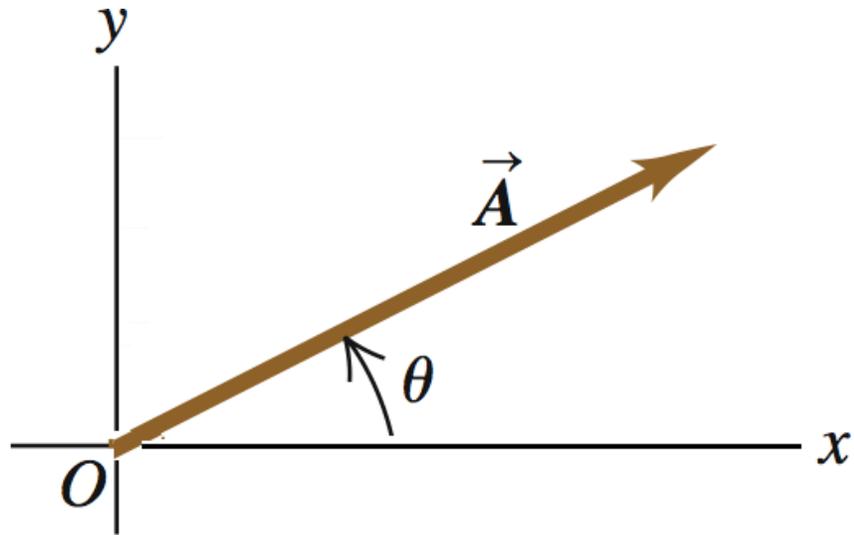
The angle θ is measured counterclockwise from the positive x -axis as shown. For which of these vectors is θ greatest?



- A. $+24\hat{i} + 18\hat{j}$
- B. $-24\hat{i} - 18\hat{j}$
- C. $-18\hat{i} + 24\hat{j}$
- D. $-18\hat{i} - 24\hat{j}$
- E. $+18\hat{i} - 24\hat{j}$

A1.12

The angle θ is measured counterclockwise from the positive x -axis as shown. For which of these vectors is θ greatest?



A. $+24\hat{i} + 18\hat{j}$

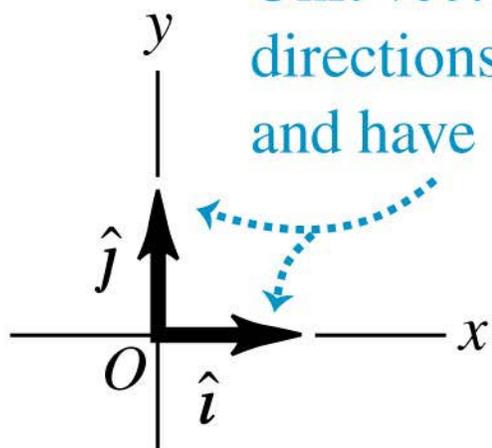
B. $-24\hat{i} - 18\hat{j}$

C. $-18\hat{i} + 24\hat{j}$

D. $-18\hat{i} - 24\hat{j}$

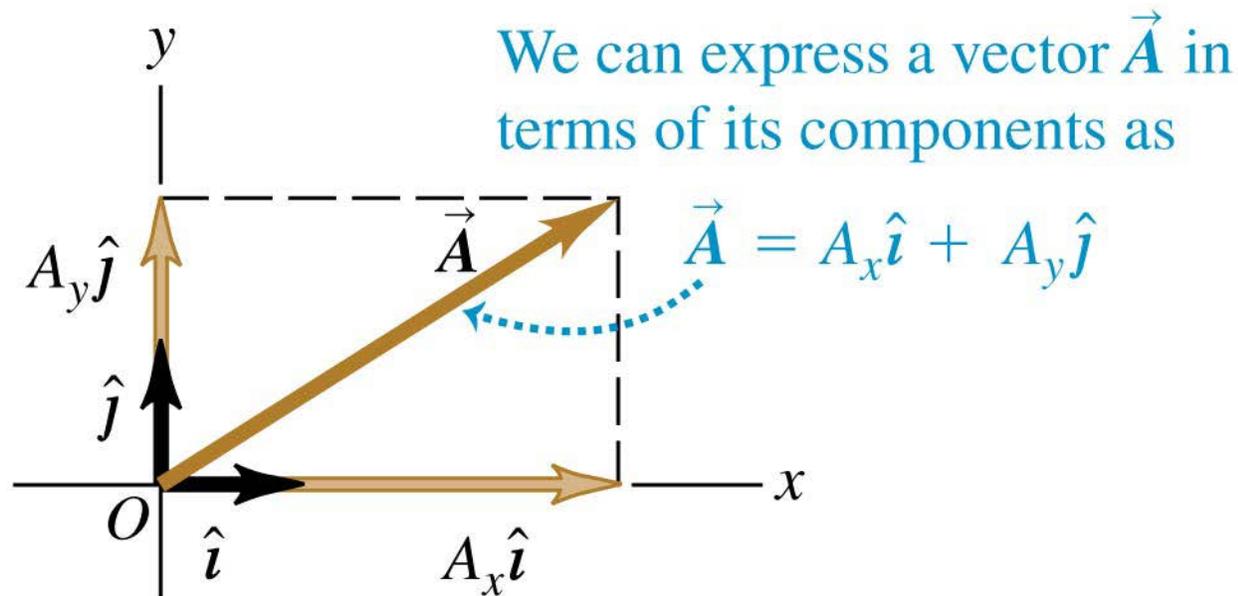
E. $+18\hat{i} - 24\hat{j}$

(a)



Unit vectors \hat{i} and \hat{j} point in the directions of the positive x - and y -axes and have a magnitude of 1.

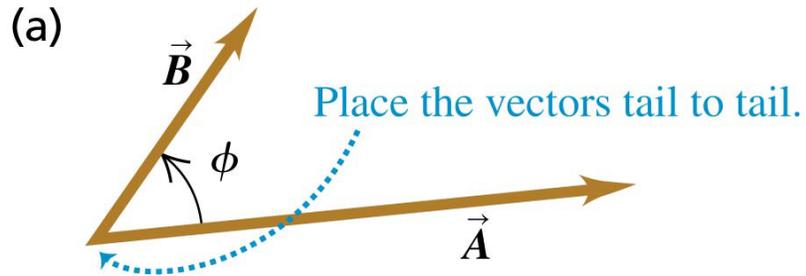
(b)



We can express a vector \vec{A} in terms of its components as

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

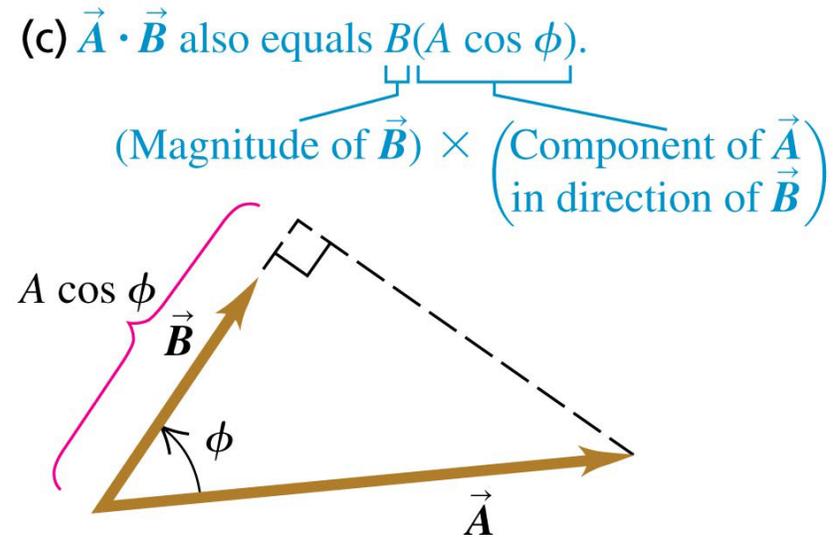
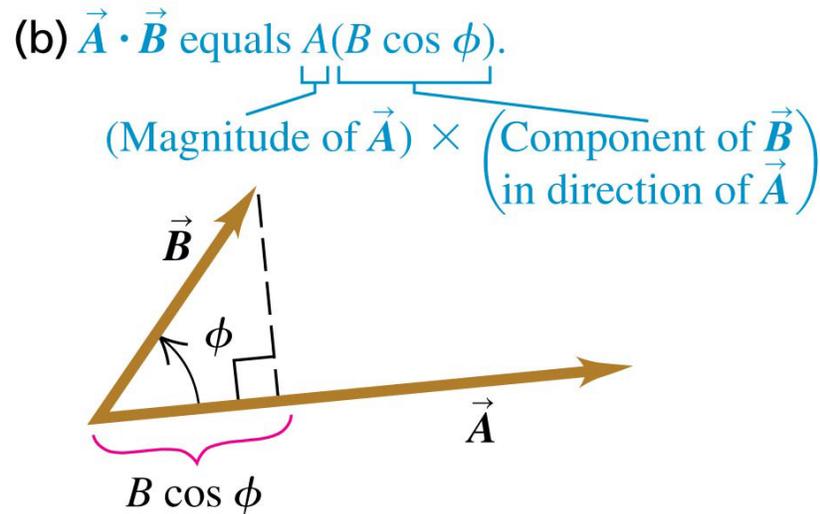
Dot Product (A.K.A. scalar product)



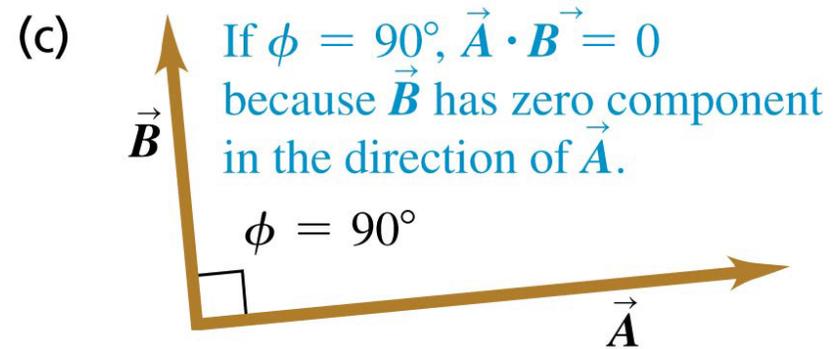
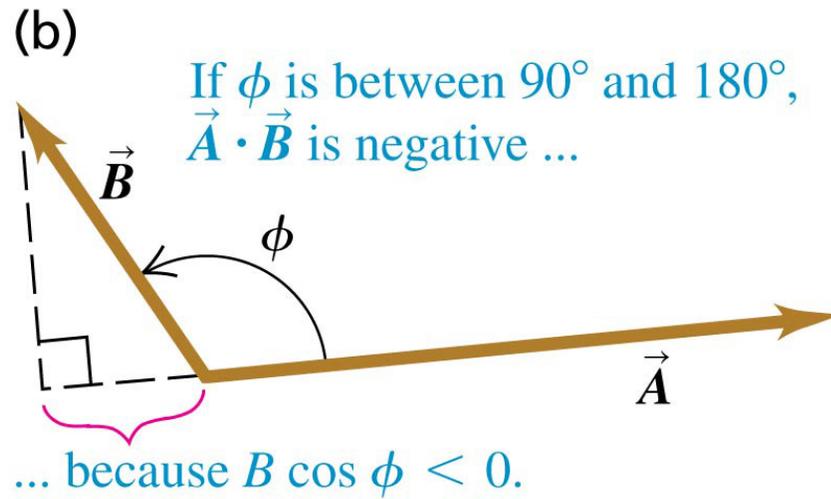
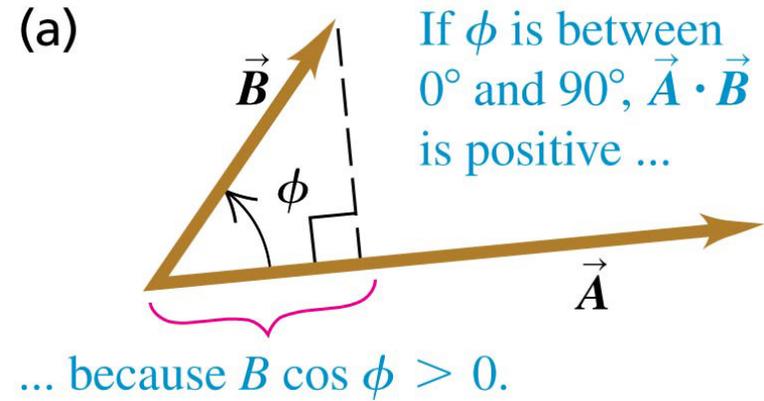
Scalar (dot) product of vectors \vec{A} and \vec{B} Magnitudes of \vec{A} and \vec{B}

$$\vec{A} \cdot \vec{B} = AB \cos \phi = |\vec{A}| |\vec{B}| \cos \phi$$

Angle between \vec{A} and \vec{B} when placed tail to tail



- The scalar product can be positive, negative, or zero, depending on the angle between \vec{A} and \vec{B} .



Calculating a Scalar Product Using Components

- In terms of components:

Scalar (dot) product
of vectors \vec{A} and \vec{B}

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

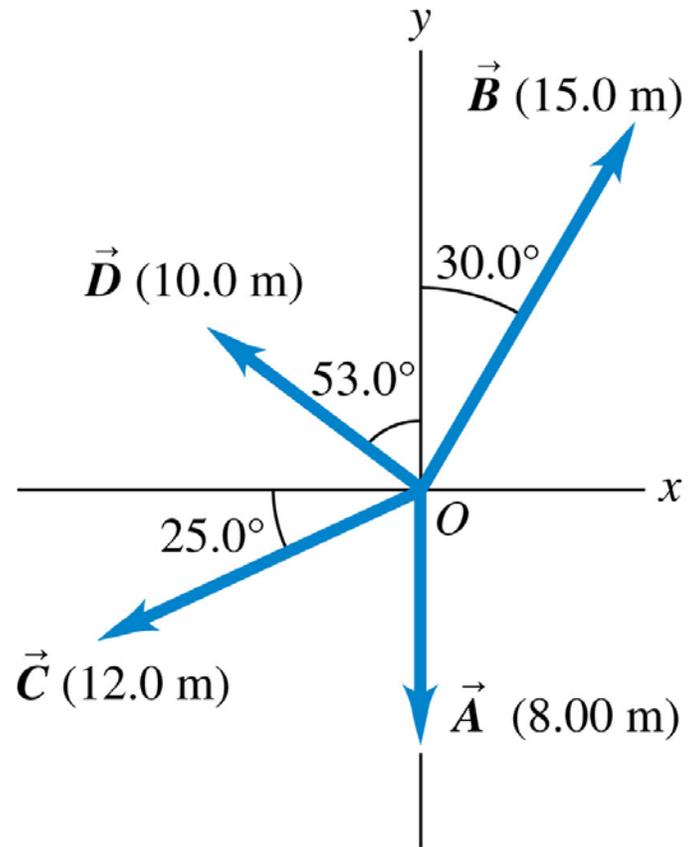
Components of \vec{A}

Components of \vec{B}

- The scalar product of two vectors is the sum of the products of their respective components.
- The angle between two vectors can be found using the dot product: $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$

Q1.13

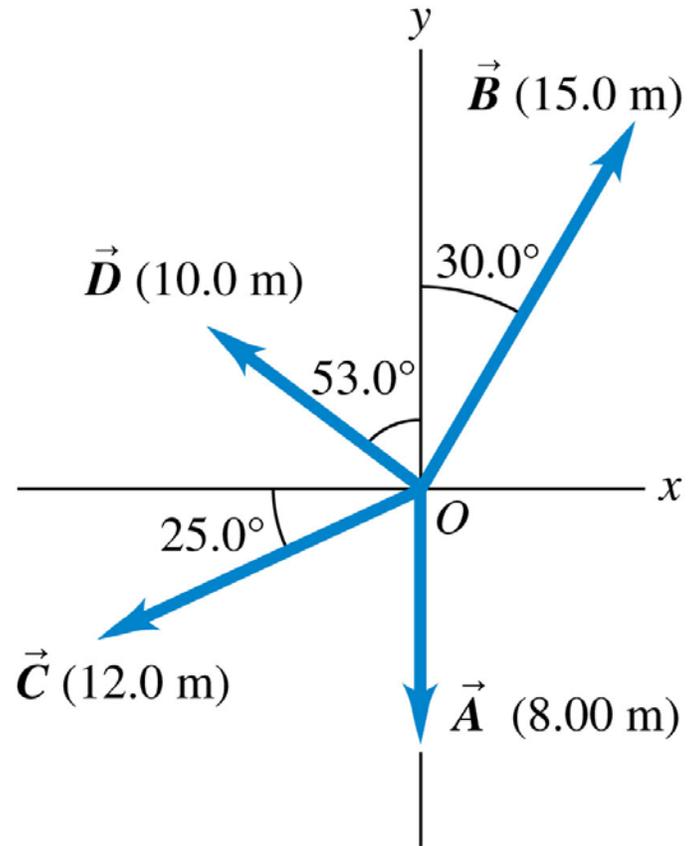
Consider the vectors shown. What is the dot product $\vec{C} \cdot \vec{D}$?



- A. $(120 \text{ m}^2) \cos 78.0^\circ$
- B. $(120 \text{ m}^2) \sin 78.0^\circ$
- C. $(120 \text{ m}^2) \cos 62.0^\circ$
- D. $(120 \text{ m}^2) \sin 62.0^\circ$
- E. none of these

A1.13

Consider the vectors shown. What is the dot product $\vec{C} \cdot \vec{D}$?



A. $(120 \text{ m}^2) \cos 78.0^\circ$

B. $(120 \text{ m}^2) \sin 78.0^\circ$

C. $(120 \text{ m}^2) \cos 62.0^\circ$

D. $(120 \text{ m}^2) \sin 62.0^\circ$

E. none of these

Q1.15

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the dot product $\vec{A} \cdot \vec{B}$?

- A. zero
- B. 14
- C. 48
- D. 50
- E. none of these

A1.15

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the dot product $\vec{A} \bullet \vec{B}$?



A. zero

B. 14

C. 48

D. 50

E. none of these

Q1.17

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the dot product $\vec{A} \cdot \vec{B}$?

- A. zero
- B. -6
- C. +6
- D. 42
- E. -42

A1.17

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the dot product $\vec{A} \cdot \vec{B}$?



A. zero

B. -6

C. +6

D. 42

E. -42

Cross Product

(A.K.A. vector product)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

For example:

$$(1 \quad 1 \quad 0) \times (-1 \quad 0 \quad 1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 1 \hat{i} - 1 \hat{j} + 1 \hat{k} = (1 \quad -1 \quad 1)$$

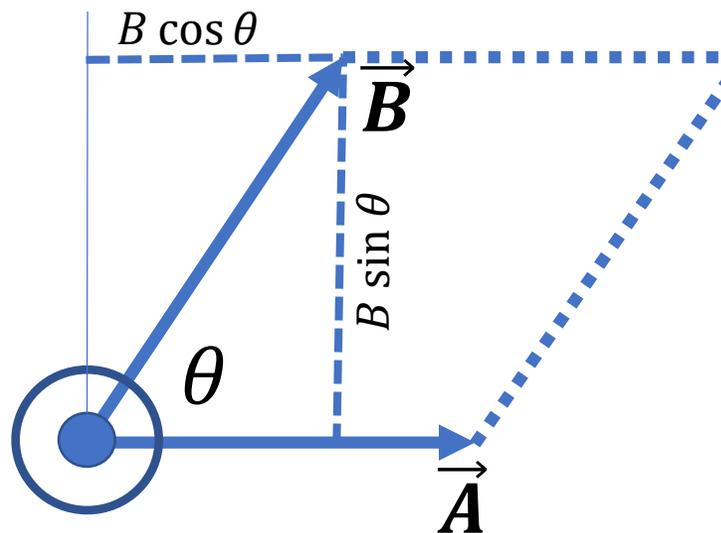
Note that the resultant vector is perpendicular to the first two.

Another example:

Cross Product (A.K.A. vector product)

Another example:

$$\begin{aligned} (|\vec{A}| \ 0 \ 0) \times (|\vec{B}| \cos \theta \ |\vec{B}| \sin \theta \ 0) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ |\vec{A}| & 0 & 0 \\ |\vec{B}| \cos \theta & |\vec{B}| \sin \theta & 0 \end{vmatrix} \\ &= |\vec{A}| |\vec{B}| \sin \theta \hat{k} = (0 \ 0 \ |\vec{A}| |\vec{B}| \sin \theta) \end{aligned}$$



If the vector product (“cross product”) of two vectors is $\vec{C} = \vec{A} \times \vec{B}$ then:

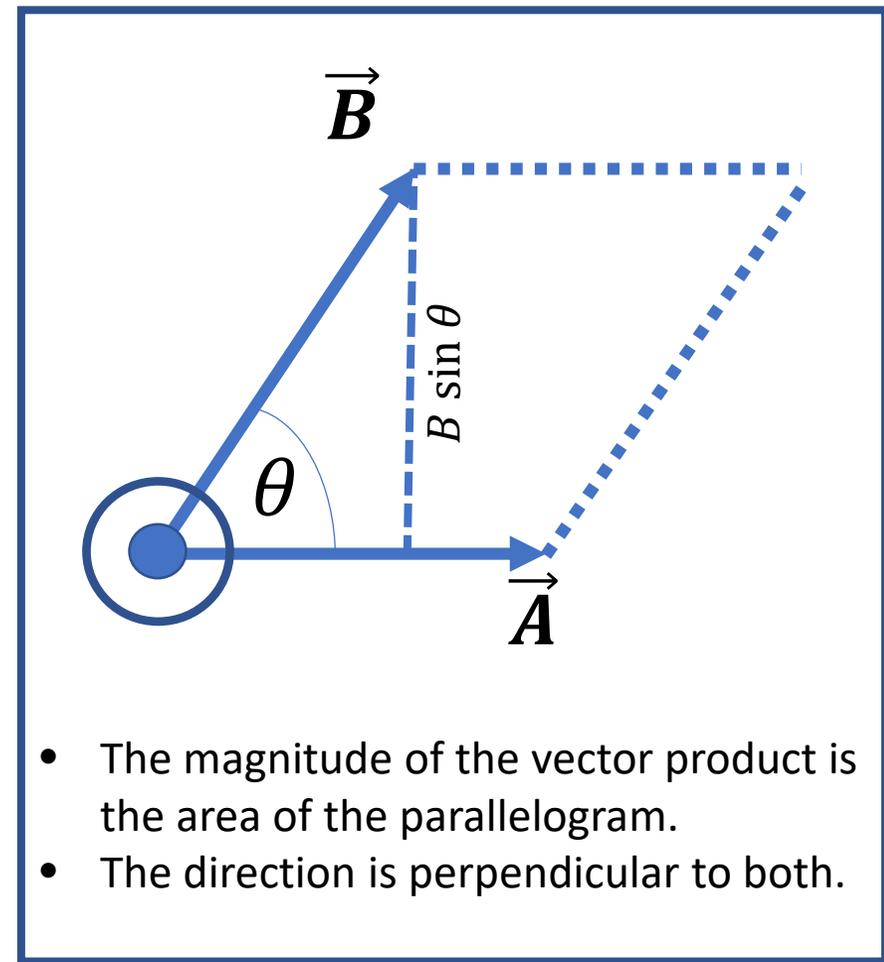
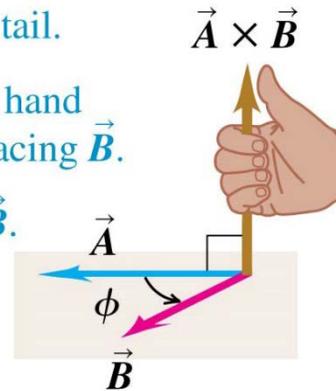
Magnitude of vector (cross) product of vectors \vec{B} and \vec{A}

$$C = AB \sin \phi$$

Magnitudes of \vec{A} and \vec{B} Angle between \vec{A} and \vec{B} when placed tail to tail

The direction of the vector product can be found using the right-hand rule:

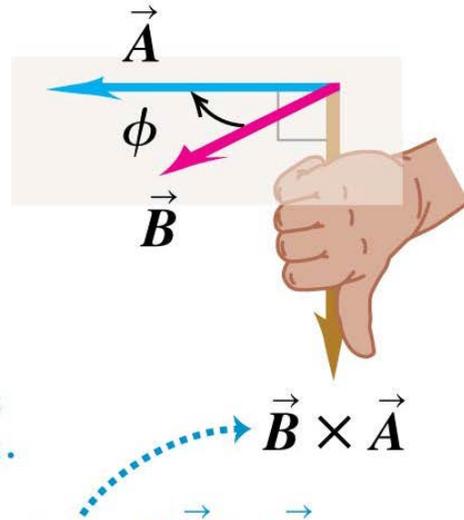
- ① Place \vec{A} and \vec{B} tail to tail.
- ② Point fingers of right hand along \vec{A} , with palm facing \vec{B} .
- ③ Curl fingers toward \vec{B} .
- ④ Thumb points in direction of $\vec{A} \times \vec{B}$.



The Vector Product is Anticommutative

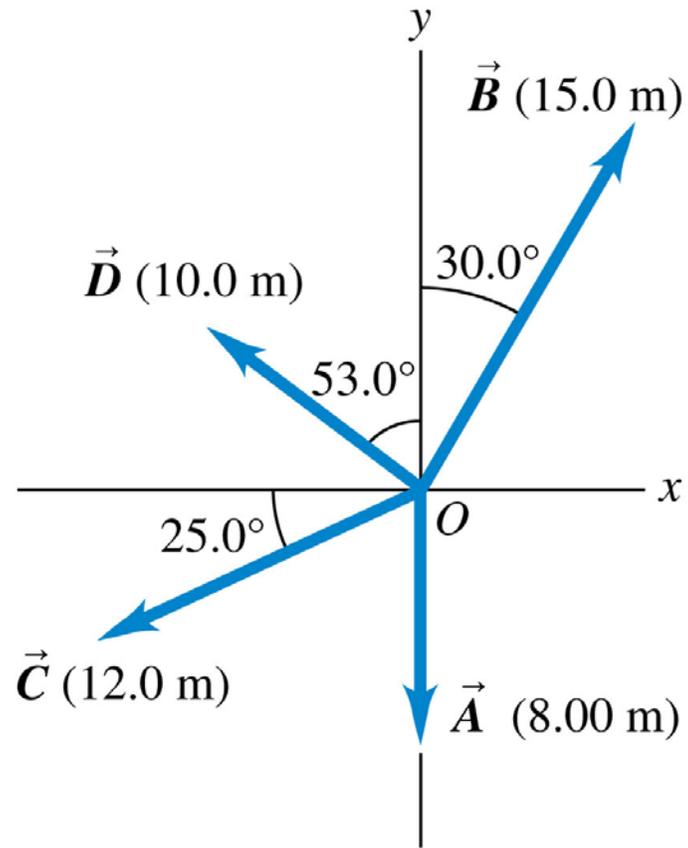
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- ① Place \vec{B} and \vec{A} tail to tail.
- ② Point fingers of right hand along \vec{B} , with palm facing \vec{A} .
- ③ Curl fingers toward \vec{A} .
- ④ Thumb points in direction of $\vec{B} \times \vec{A}$.
- ⑤ $\vec{B} \times \vec{A}$ has same magnitude as $\vec{A} \times \vec{B}$ but points in opposite direction.



Q1.14

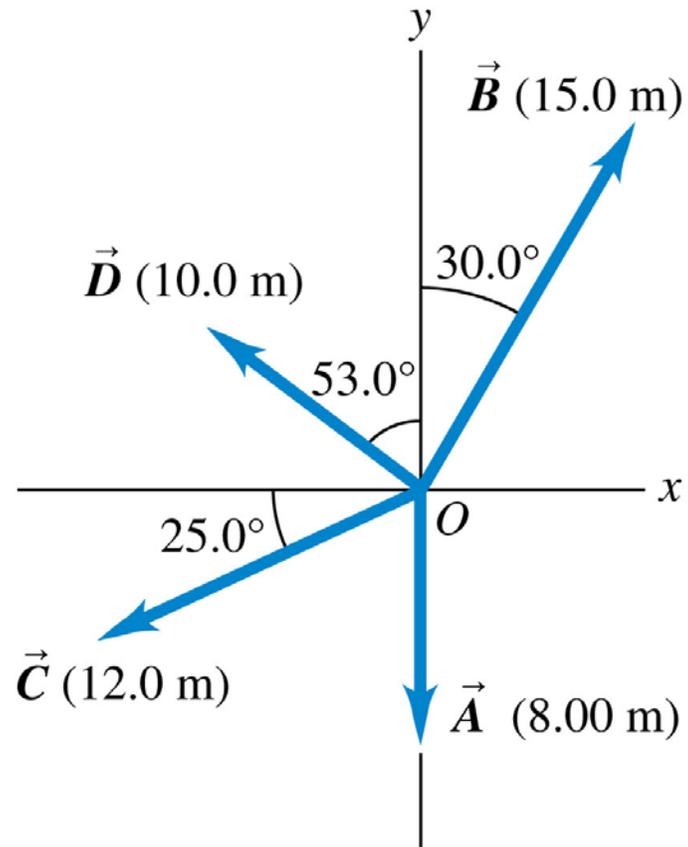
Consider the vectors shown. What is the cross product $\vec{A} \times \vec{C}$?



- A. $(96.0 \text{ m}^2) \sin 25.0^\circ \hat{k}$
- B. $(96.0 \text{ m}^2) \cos 25.0^\circ \hat{k}$
- C. $-(96.0 \text{ m}^2) \sin 25.0^\circ \hat{k}$
- D. $-(96.0 \text{ m}^2) \cos 25.0^\circ \hat{k}$
- E. none of these

A1.14

Consider the vectors shown. What is the cross product $\vec{A} \times \vec{C}$?



A. $(96.0 \text{ m}^2) \sin 25.0^\circ \hat{k}$

B. $(96.0 \text{ m}^2) \cos 25.0^\circ \hat{k}$

C. $-(96.0 \text{ m}^2) \sin 25.0^\circ \hat{k}$

D. $-(96.0 \text{ m}^2) \cos 25.0^\circ \hat{k}$

E. none of these

Q1.16

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. $6\hat{k}$

B. $-6\hat{k}$

C. $50\hat{k}$

D. $-50\hat{k}$

E. none of these

A1.16

Consider the two vectors

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -8\hat{i} + 6\hat{j}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. $6\hat{k}$

B. $-6\hat{k}$

 C. $50\hat{k}$

D. $-50\hat{k}$

E. none of these

Q1.18

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. zero

B. $24\hat{i} + 18\hat{j}$

C. $-24\hat{i} - 18\hat{j}$

D. $-18\hat{i} + 24\hat{j}$

E. $-18\hat{i} - 24\hat{j}$

A1.18

Consider the two vectors

$$\vec{A} = 3\hat{i} - 4\hat{j}$$

$$\vec{B} = 6\hat{k}$$

What is the cross product $\vec{A} \times \vec{B}$?

A. zero

B. $24\hat{i} + 18\hat{j}$

 C. $-24\hat{i} - 18\hat{j}$

D. $-18\hat{i} + 24\hat{j}$

E. $-18\hat{i} - 24\hat{j}$